Research Internship Report

Generalized Global Symmetries in QFT

Author:
Md. Samim-ul-Islam

Supervisor:
Dr. Arpit Das

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1 Introduction

When we started studying physics, we were interested in understanding microscopic constituents, which are the fundamental building blocks of nature. Quantum field theory remains the most effective language of these fundamental building blocks to date. It is a powerful mathematical framework that has revolutionized our understanding of the fundamental forces of nature, from the subatomic scale to the cosmological scale. Its applications are huge, ranging from high energy physics to condensed matter physics. One of the most significant achievements of QFT has been the unification of the electromagnetic force and the weak nuclear force of particle physics, as well as the Higgs mechanism. The key role played in this context is the use of symmetries, which simplify the problem by reducing the number of degrees of freedom or by mapping the problem to another, more tractable system.

However, there are many challenges in QFT, such as understanding the behavior of a system with strong coupling, among others. Besides these challenges, there are some seemingly mundane but nuanced questions in QFT, specifically Quantum Electrodynamic (QED), that are of particular interest in this report, such as

Q. What is the symmetry principle behind the conservation of magnetic flux lines?

In ordinary 4D Maxwell theory, there are magnetic flux lines (e.g., magnetic field lines of a solenoid) that do not end in space or time due to the non-existence of magnetic monopoles. So, the number of these field lines is a conserved quantity. This is a result of the Bianchi identity, also known as Gauss law for magnetostatics and it's an off-shell conservation as seen in the electric frame. But using S-duality (see Appendix A), we can go to the magnetic frame. In this frame, Gauss law for magnetostatics becomes the equation of motion, and according to Noether's Theorem, there must be some symmetry associated with it.

Q. Why photons are massless in D = 4?

The most common answer would be $U(1)$ gauge invariance. However, if we look at Anderson's work [1], we see that photons can be gauge-invariant and still be massive!!

Q. How can we extend the Landau Paradigm to explain phenomena, e.g., topological order, non-Fermi liquids, etc., under the framework of QFT?

According to the Landau Paradigm, *different phases of matter are defined by patterns of* ***broken & unbroken symmetries*** *and critical points between phases can be studied by* ***universal theories*** *of the* ***order parameter***.
However, much of the modern work of condensed matter involving phases and transitions in between does not fit into this paradigm, e.g., we have the Coulomb and Higgs phases $U(1)$ gauge theory in the continuum, which seem different because there is no local order parameter for this, and similarly for non-Abelian gauge theory and confinement; topological order (for instance, deconfined phases of lattice gauge theory, fractional quantum Hall states, etc.); non-Fermi liquids; deconfined criticality; etc.

It turns out that the symmetry principle behind these questions has only recently been understood [2], motivated by non-Abelian gauge theory. These generalized notions of symmetries help in recasting various properties of gauge theories and also lead to the discovery of many phases of the theories. These symmetry structures are generalized in a way that they do not possess a group structure; rather, they have a higher group (in some cases of higher-form symmetries) or categorical structure (in the case of non-invertible symmetries), where some of the group axioms are relaxed [3, 4, 5].

The objective of this report is to explore these generalized notions of symmetries and, in part, to explain the questions mentioned above under the framework of generalized symmetries.

2 0-Form Symmetries

Symmetries play a crucial role in understanding the behavior of any physical system. They are a powerful tool for simplifying and solving problems in theoretical physics. In this section, we will have a brief review of some basic aspects of 0-form, i.e., ordinary symmetries in QFT, which will be useful in the generalization to the case of higher-form symmetries.

To orient the discussion on symmetries, let's start with an example of $U(1)$ 0-form symmetry. Such a symmetry is associated to a Noether current, which is a local operator $J^\mu(x)$ satisfying the continuity equation [6]

$$\partial_\mu J^\mu(x) = 0 \tag{1}$$

In terms of differential forms, the Noether current can be thought of as the components of a 1-form,

$$J = J_\mu dx^\mu \tag{2}$$

whereas its Hodge dual,

$$\star J = \frac{1}{(D-1)!} J_\mu \epsilon^\mu_{\nu_1 \cdots\cdots \nu_{D-1}} dx^{\nu_1} \wedge dx^{\nu_2} \wedge \cdots\cdots \wedge dx^{\nu_{D-1}} \tag{3}$$

is a $(D-1)$-form.

As the current is conserved on-shell, this means that if ϕ satisfies the equations of motion, we get

$$d \star J = 0 \tag{4}$$

and hence the Noether charge can be written as an integral over a codimension-1 submanifold Σ_{D-1}

$$Q(\Sigma_{D-1}) = \int_{\Sigma_{D-1}} \star J \tag{5}$$

2.1 Ward Identities

The classical conservation equation 1 is somewhat modified in the quantum theory, which is the so-called Ward identities, which lead to relations among correlation functions of the theory [6]. They can be derived in a simple way using the path integral [3, 6]. Consider the partition function

$$Z = \int \mathcal{D}\phi e^{iS} \tag{6}$$

So, correlation functions are expressed as

$$\langle X \rangle = \frac{1}{Z} \int \mathcal{D}\phi X e^{iS} \;;\; X = \prod_{j=1}^{N} \phi(x_j) \tag{7}$$

As the fields are merely integration variables, we are free to rename them or to make changes to the integration variables. First, we rename the fields $\phi \to \phi'$ and then perform a variable change in the path integral according to

$$\phi \to \phi + \epsilon(x)\delta\phi \tag{8}$$

This amounts to

$$\begin{aligned}
\langle X \rangle &= \frac{1}{Z} \int \mathcal{D}\phi \prod_{j=1}^{N} \phi(x_j) e^{iS} \\
&= \frac{1}{Z} \int \mathcal{D}\phi' \prod_{j=1}^{N} \phi'(x_j) e^{iS'} \\
&= \frac{1}{Z} \int \mathcal{D}\phi [X + \sum_j \phi(x_1).....\epsilon_a \delta\phi_a(x_j).....\phi(x_N)] e^{iS} e^{i\delta S} \\
&= \frac{1}{Z} \int \mathcal{D}\phi [X + \sum_j \phi(x_1).....\epsilon_a \delta\phi_a(x_j).....\phi(x_N)](1 + i\delta S) e^{iS} \\
&= \frac{1}{Z} \int \mathcal{D}\phi [X + \sum_j \phi(x_1).....\epsilon_a \delta\phi_a(x_j).....\phi(x_N) + iX\delta S + \mathcal{O}(\epsilon^2)] e^{iS} \\
\Rightarrow 0 &= \int d^D x \epsilon_a(x) [\sum_j \delta^{(D)}(x - x_j) \langle \phi(x_1).....\delta\phi_a(x_j).....\phi(x_N) \rangle - i\partial_\mu \langle J_a^\mu(x) X \rangle + \mathcal{O}(\epsilon^2)]
\end{aligned} \tag{9}$$

where, $\delta S = \int d^D x J_a^\mu(x) \partial_\mu \epsilon_a(x) = -\int d^D x \epsilon_a(x) \partial_\mu J_a^\mu(x)$. Thus, the above relation enables us to find the Ward identities

$$\partial_\mu \langle J_a^\mu(x) \phi(x_1)....\phi(x_N) \rangle = -i \sum_j \delta^{(D)}(x - x_j) \langle \phi(x_1).....\delta\phi_a(x_j).....\phi(x_N) \rangle \tag{10}$$

which provides a set of relations among correlation functions and conservation laws at non-coincident points.

Now, Integrating x over the region $[t_+, t_-] \times \mathbb{R}^{D-1}$

$$\begin{aligned} \langle [Q(t_+) - Q(t_-)]\phi(x_1)Y\rangle &= -i\langle \delta\phi(x_1)Y\rangle \\ \Rightarrow \langle (Q(t_+)\phi(x_1) - \phi(x_1)Q(t_-))Y\rangle &= -i\langle \delta\phi(x_1)Y\rangle \\ \Rightarrow \langle [Q(t), \phi(x_1)]Y\rangle &= -i\langle \delta\phi(x_1)Y\rangle \\ \Rightarrow [Q(t), \phi(x_1)] &= -i\delta\phi(x_1) \end{aligned} \tag{11}$$

Here, $Y = \prod_{j=2}^{N} \phi(x_j)$

2.2 Phases of Matter, SSB & Goldstone Mode

Recall that under ordinary, i.e., 0-form symmetry, the field ϕ is the order parameter for the transition [8], because it behaves so differently in the two phases:

- **Unbroken Phase:** For $\mu^2 > 0$, we have a unique vacuum in which ϕ is basically zero, i.e., $\langle\phi(x)\rangle = 0$. Because this vacuum is invariant under, say, U(1) symmetry, thus U(1) symmetry is unbroken. In this phase, we can imagine doing a calculation of the following two-point function:

$$\langle \phi^\dagger(x)\phi(y)\rangle \sim e^{-m|x-y|} \tag{12}$$

- **SSB Phase:** For $\mu^2 < 0$, the potential has a family of minima at $|\phi|^2 = v^2$. Thus the vacuum will have a nonzero value of $\langle\phi(x)\rangle = |v|$. In this phase, the particles have condensed; thus, the U(1) symmetry is spontaneously broken. Hence, consider the computation of the following two-point function:

$$\lim_{|x-y|\to 0} \langle \phi^\dagger(x)\phi(y)\rangle \sim \langle \phi^\dagger(x)\rangle\langle\phi(y)\rangle \tag{13}$$

$$\text{When } |\mu|^2 = \begin{cases} \mu^2 > 0, & \langle\phi\rangle_0 = 0 \\ \mu^2 < 0, & \langle\phi\rangle_0 = |v| \end{cases}$$

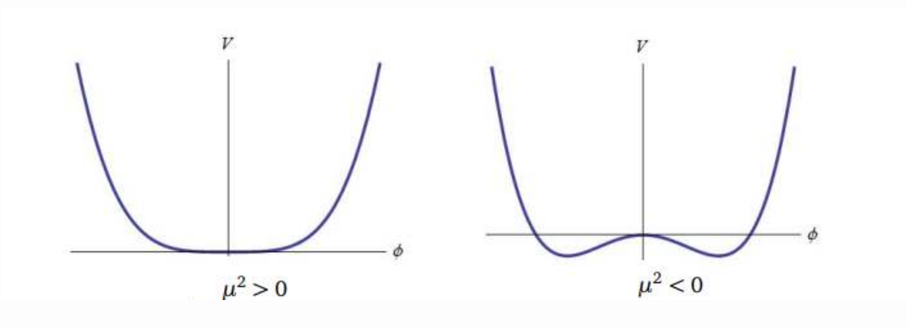

Figure 1: Two phase of $U(1)$ symmetry

Now, we will derive the most important consequence of SSB, i.e., the Goldstone theorem, which states that when a global continuous symmetry is spontaneously broken, then there are massless excitations (Goldstone bosons) in the spectrum [7]. This result can be directly constructed from the Ward identity 10. For a single field,

$$\partial_\mu \langle J^\mu_a(x)\phi_a(y)\rangle = -i\delta^{(D)}(x-y)\langle\delta\phi_a(y)\rangle \tag{14}$$

Taking the Fourier transform with respect to x

$$\begin{gathered}\int d^D x e^{ipx}\partial_\mu\langle J^\mu_a(x)\phi_a(y)\rangle = -i\int d^D x e^{ipx}\delta^{(D)}(x-y)\langle\delta\phi_a(y)\rangle \\ => -i\int d^D x e^{ipx}p_\mu\langle J^\mu_a(x)\phi_a(y)\rangle = -ie^{ipy}\langle\delta\phi_a(y)\rangle \\ => p_\mu\langle J^\mu_a(p)\phi_a(y)\rangle = e^{ipy}\langle\delta\phi_a(y)\rangle \ ; \ J^\mu_a(p) = \int d^D x e^{ipx}J^\mu_a(x) \\ => p_\mu\langle J^\mu_a(p)e^{-ipy}\phi_a(y)\rangle = \langle\delta\phi_a(y)\rangle\end{gathered} \tag{15}$$

Integrating over y both sides

$$p_\mu\langle J^\mu_a(p)\phi_a(-p)\rangle = \int d^D y\langle\delta\phi_a(y)\rangle = \langle\delta\phi_a(p=0)\rangle \tag{16}$$

Since $\langle\delta\phi_a(p=0)\rangle$ is the order parameter that characterizes the possible phases of the theory [8] that is

$$\langle\delta\phi_a(p=0)\rangle = \begin{cases} 0, & Unbroken\ phase \\ otherwise, & Broken\ phase\end{cases} \tag{17}$$

In the broken phase, therefore, the correlation function $\langle J^\mu_a(p)\phi_a(-p)\rangle$ must have a pole at zero momentum. To see this from above, first note that $\langle J^\mu_a(p)\phi_a(-p)\rangle \sim c(p)p^\mu$ (obtained by index structure of current).

Now, in the case of an SSB phase, $\langle\delta\phi_a(p=0)\rangle \neq 0$ is some constant independent of the momentum p^μ. To satisfy this we must have $c(p) \sim \frac{1}{p^2}$ i.e.

$$\langle J^\mu_a(p)\phi_a(-p)\rangle \sim \frac{p^\mu}{p^2} \tag{18}$$

This indicates the presence of massless physical excitations within the spectrum. Such excitations are referred to as Goldstone bosons.

2.3 Charge Operator as Topological Operator

The physical interpretation of Noether charge

$$Q(\Sigma_{D-1}) = \int_{\Sigma_{D-1}} \star J \tag{19}$$

is that it is the integral over the time-slice intersects all of the particle world lines – you can move it up and down in time and get the same answer [5]. Moreover, the world-lines can't end because the particle number is conserved, which is just the count of the particle number.

Notice that, $Q(\Sigma_{D-1})$ appears to be a function of the manifold Σ_{D-1}– however, because the current J^μ is conserved, it is actually independent under small wiggles of Σ_{D-1}: it is a topological defect in the theory (topological means invariant under deformations of some manifold) which is we will see below [3]:

Using Stokes theorem on Ward Identity for a single field 14, we get

$$\int_{\Sigma_{D-1}} \langle d \star J\phi(y)\rangle = \int_{\partial\Sigma_{D-1}} \langle \star J\phi(y)\rangle = \langle Q(\Sigma)\phi(y)\rangle \tag{20}$$

According to 14,

$$\langle Q(\Sigma_{D-1})\phi(y)\rangle = -i\int_{\Sigma_{D-1}} d^D x\delta^{(D)}(x-y)\langle\delta\phi(y)\rangle \tag{21}$$

So, we can identify $\int_\Sigma d^D x\delta^{(D)}(x-y)$ as intersection number of Σ and y. Therefore,

$$\text{Link}(\Sigma_{D-1}, y) = \int_{\Sigma_{D-1}} d^D x\delta^{(D)}(x-y) \tag{22}$$

which is 0 or 1, depending on whether y is inside the region Σ or not. Hence, we write,

$$\langle Q(\Sigma_{D-1})\phi(y)\rangle = -i\text{Link}(\Sigma_{D-1}, y)\langle\delta\phi(y)\rangle \tag{23}$$

Now, note that both Link number defined in 22 and charge, $Q(\Sigma_{D-1})$ are topological. So we consider a deformation of the original region Σ_{D-1} to $\Sigma'_{D-1} = \Sigma_{D-1}\cup\Sigma_0$, such that $y \notin \Sigma_0$. This implies

$$\begin{aligned} Q(\Sigma'_{D-1}) = \int_{\Sigma'_{D-1}} \langle d\star J\phi(y)\rangle &= \int_{\Sigma'_{D-1}} \langle d\star J\phi(y)\rangle + \int_{\Sigma_0} \langle d\star J\phi(y)\rangle \\ &= \int_{\partial\Sigma_{D-1}} \langle\star J\phi(y)\rangle = Q(\Sigma_{D-1}) \end{aligned} \tag{24}$$

where, as $y \notin \Sigma_0$, we have set $d \star J = 0$ inside the correlation in the last term of the first line. Therefore, conservation law is translated into the fact that the charge operator, $Q(\Sigma_{D-1})$ is topological.

For continuous symmetries, the unitary operator implementing the corresponding transformation can be systematically constructed from this Noether charge

$$U(\Sigma_{D-1}) = e^{i\alpha\int_{\Sigma_{D-1}}\star J} \tag{25}$$

whose properties are as follows [4]:

- **Topological Nature :** U is a topological operator i.e.

$$U(\Sigma_{D-1}) = U(\Sigma'_{D-1}) \tag{26}$$

This implies that codimension-1 submanifold of spacetime, Σ'_{D-1} can be obtained by topologically deforming Σ_{D-1} (Figure 2).

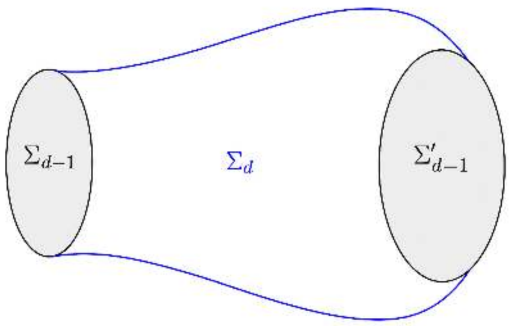

Figure 2: Codimension-1 submanifold of spacetime, Σ'_{D-1} can be obtained by topologically deforming Σ_{D-1}.Placing a topological operator U along Σ_{D-1} is equivalent to placing it along Σ'_{D-1}.

Let's get back to the example of $U(1)$ 0-form symmetry to unpack this further: Since Σ'_{D-1} can be obtained by topologically deforming Σ_{D-1} i.e., $\Sigma'_{D-1} - \Sigma_{D-1} = \partial\Sigma_D$ we can write

$$\begin{aligned} U(\Sigma'_{D-1}) &= U(\Sigma_{D-1}) \times e^{i\alpha\int_{\partial\Sigma_D} \star J} \\ &= U(\Sigma_{D-1}) \times e^{i\alpha\int_{\Sigma_D} d\star J} \\ &= U(\Sigma_{D-1}) \end{aligned} \tag{27}$$

Recall canonical QFT, this is just an analogous way of saying that a unitary operator, $U(t)$, acting on the Hilbert space at time t commutes with the Hamiltonian generating time evolution, i.e.

$$U(t) = U(t') \;;\; \forall t, t' \tag{28}$$

- **Invertibility :** U is invertible i.e.

$$U(\Sigma_{D-1})U'(\Sigma_{D-1}) = \mathbb{1} \tag{29}$$

This means that there exists another topological codimension-1 operator U' such that inserting both U and U' along the same codimension-1 submanifold Σ_{D-1} is equivalent to inserting no operator along Σ_{D-1}.

- **Group multiplication law :** Group multiplication describes the composition of these operators
$$U(\Sigma_{D-1})U'(\Sigma_{D-1}) = e^{i(\alpha+\alpha')\int_{\Sigma_{D-1}} \star J} \tag{30}$$

- Action on Local Operator : U acts on the local operator $\mathcal{O}(x)$ by converting it into the local operator $\mathcal{O}'(x)$ i.e.
$$U(\Sigma_{D-1})\mathcal{O}(x) = \mathcal{O}'(x)U(\Sigma'_{D-1}) \ ; \ x \equiv (\vec{x}, t) \tag{31}$$
which is analogous to $U(t)$ acts on all of space, because it acts on local operators located at any point in space. The action is implemented by conjugation
$$U(t)\mathcal{O}(\vec{x}, t)U^{-1}(t) = \mathcal{O}'(\vec{x}, t) \tag{32}$$
Let's get back to the example of $U(1)$ 0-form symmetry to unpack this further: According to the Ward identity, the current is not conserved in the presence of a local operator $\mathcal{O}(x)$ of charge $q \in \mathbb{Z}$ under $U(1)$, i.e., we have a modification to the continuity equation
$$\mathcal{O}(x) d \star j = q\delta^D(x)\mathcal{O}(x) \tag{33}$$
where $\delta^D(x)$ is the D-form Poincaré dual to the delta function $\delta(x - x')$. Since for any D-dimensional spacetime, a point can always link with a closed $(D-1)$-dimensional manifold like S^{D-1} that surrounds it (Figure 3). Hence,
$$\begin{aligned} U(S^{D-1})\mathcal{O}(x) &= e^{i\alpha \int_{\partial\Sigma_D} \star J}\mathcal{O}(x) \\ &= e^{i\alpha \int_{\Sigma_D} d\star J}\mathcal{O}(x) \\ &= e^{i\alpha \int_{\Sigma_D} q\delta^D(x)}\mathcal{O}(x) \\ &= e^{i\alpha q}\mathcal{O}(x) \end{aligned} \tag{34}$$
So, $\mathcal{O}(x)$ will pick up a phase. This is most cleanly represented if we imagine placing the defect on a S^{D-1} that links the $\mathcal{O}(x)$ and then collapsing it (Figure 3).

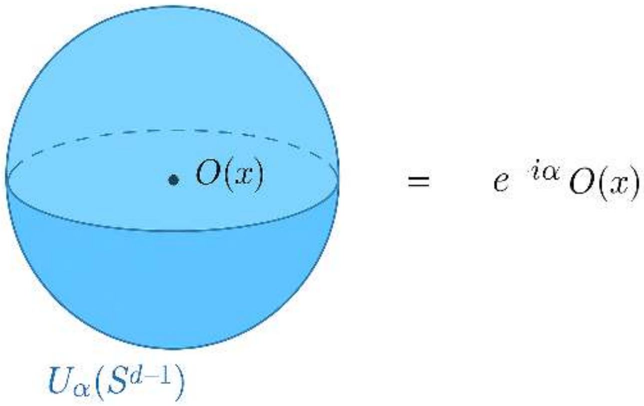

Figure 3: Collapsing $U(S^{D-1})$ into local operator $\mathcal{O}(x)$ acts as phase rotation.

Notice that throughout the whole subsection, we have constructed the same old canonical QFT in a generalized notion. Seeing such fashion, a crucial insight about 0-form symmetry [2] can be expressed as follows:

> "0-*form symmetries are Topological codimension-1 operators which are invertible.*"

3 Higher Form Symmetries

In this section, we will generalize the previous discussion to the case of a 1-form symmetry.

Like before, we will start with Noether's current. For 1-form symmetry, the current can be thought of as the components of a 2-form

$$J = \frac{1}{2!} J_{\mu\nu} dx^\mu \wedge dx^\nu \tag{35}$$

Hence, the conservation law $d \star J = 0$ reads

$$\partial_\mu J^{\mu\nu} = 0 \; ; \; J^{\mu\nu} = J^{[\mu\nu]} \tag{36}$$

Note that, we are choosing antisymmetric tensors because they don't need a metric to define parts of the action and hence can describe topological objects, while symmetric tensors like the stress tensor couple to the metric by definition and hence can't describe topological objects. Now, Noether's charge

$$Q(\Sigma_{D-2}) = \int_{\Sigma_{D-2}} \star J \tag{37}$$

Like 0-form global symmetries count particles, this object is associated with a $U(1)$ symmetry, which counts strings! Because strings are one-dimensional extended objects that do not end in space or in time. So, an integral over a codimension-2 surface is enough to count all the strings.

Note that, the 2-form conserved current in 36 has one extra index compared to that of the usual 1-form current associated to conventional 0-form symmetries. This extra index can be intuitively understood as referring to the direction in which a string points in spacetime.

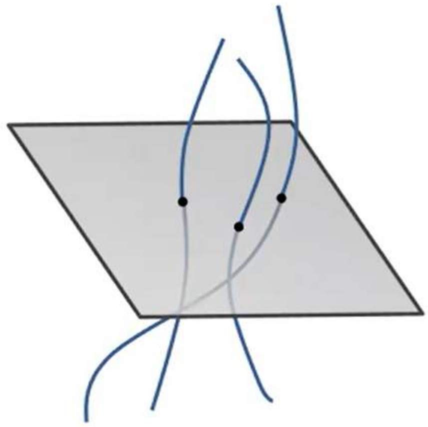

Figure 4: Integration over Σ_{D-2} counts the number of strings that crosses it at a given moment.

For 1-form symmetries, the unitary operator can be constructed from the Noether charge

$$U(\Sigma_{D-2}) = e^{i\alpha \int_{\Sigma_{D-2}} \star J} \tag{38}$$

whose properties are as follows [4]:

- **Topological Nature :** U is a topological operator i.e.

$$U(\Sigma_{D-2}) = U(\Sigma'_{D-2}) \tag{39}$$

- **Invertibility :** U is invertible i.e.

$$U(\Sigma_{D-2})U'(\Sigma_{D-2}) = \mathbb{1} \tag{40}$$

- **Group multiplication law :** Group multiplication describes the composition of these operators

$$U(\Sigma_{D-2})U'(\Sigma_{D-2}) = e^{i(\alpha+\alpha') \int_{\Sigma_{D-2}} \star J} \tag{41}$$

Another thing to mention is that *higher form symmetries are always abelian* because we can use topological deformations to change the ordering of two topological operators of codimension greater than one.

3.1 Action on Extended Operator

From 0-form symmetry, we learnt that they act on a 0-dimensional point-like local operator. This action can be understood as a consequence of moving the codimension-1 topological operator $U(\Sigma_{D-1})$ associated to the 0-form symmetry past the local operator (Figure 3).

So now we ask, *On what these higher form symmetries act on?* Generalizing the notion of 0-form, the answer is $p \geq 1$-form symmetry does not act on any local operators, as one can deform the corresponding codimension- $(p+1)$ topological operator $U(\Sigma_{D-p-1})$ past a local operator $\mathcal{O}(x)$ along a codimension-p submanifold Σ_{D-p} that does not intersect the point x [4] and that's how they also bypass the powerful Coleman-Mandula Theorem [9], which is a No-go theorem.

Coleman–Mandula Theorem (No-go Theorem)

Sarcastically, *If we try to generalise, say the current of a theory by adding more indices (e.g., higher spin objects like $J^{(\mu\nu\rho)}$), then Coleman & Mandula proved that any QFT in $D > 2$ that has such currents is necessarily a theory of free particles, i.e., S-matrix is trivial !!*

Intuitively, the reason behind this is that if you consider higher spin currents, then the corresponding conserved charges would have to be a nonlinear polynomial of momenta of the particles, and they would lead to conservation laws involving sums of squares and cubes of the external particle momenta. And if we impose such conservation laws, they are extremely powerful and way over-constrained. These over-constraints render the scattering problem intractable, making scattering impossible.

Hence, Higher form symmetries do not act on particles and local operators, and thus they evade the No-go Theorem.

Thus, from the above discussion, we can state in terms of Link:

p-dimensional object can link with q-dimensional object in D-dimension if $D = p + q + 1$ [13].

So, for 1-form symmetry, the unitary operator, $U(\Sigma_{D-2})$ acts on line operator, $W[\mathcal{C}]$ as we can link the corresponding codimension-2 topological operator with 1-dimentional line operator (Figure 5) and the corresponding equation similar to 34 be

$$U(\Sigma_{D-2})W[\mathcal{C}] = e^{-i\text{Link}(\Sigma_{D-2},\mathcal{C})}W[\mathcal{C}] \tag{42}$$

where, $\text{Link}(\Sigma_{D-2}, \mathcal{C}) = \int_{\Sigma_{D-2}} \delta^{(D-1)}(x \in \mathcal{C})$ similar to 22.

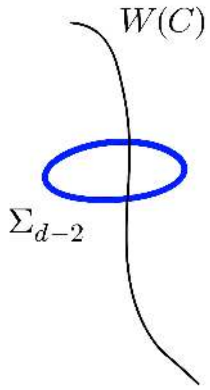

Figure 5: $U(\Sigma_{D-2})$ links the line operator, $W[\mathcal{C}]$ and then collapsing it, we see phase rotation in $U(1)$ 1-form symetry.

Therefore, from the above discussion on 1-form symmetries, it can even be generalized to any p-form symmetry [2] as follows:

"p-form symmetries are Topological codimension-$(p+1)$ operators which are invertible."

3.2 Ward Identities

We remember the case of a 0-form symmetry, where under an infinitesimal global transformation characterized by the closed 0-form parameter, ϵ, i.e., $d\epsilon = 0$. Hence, a local operator transforms as

$$\phi(x) \to \phi'(x) = \phi(x) + \epsilon\delta\phi(x) \tag{43}$$

Likewise, the infinitesimal transformation for 1-form symmetry reads [3]

$$W(\mathcal{C}) \to W'(\mathcal{C}) = W(\mathcal{C}) + \int_{\mathcal{C}} \xi(\Sigma_{D-2})\delta W(\mathcal{C}) \tag{44}$$

Here, the transformation parameter is a closed 1-form $\xi_1 = \xi_\mu dx^\mu$, namely, $d\xi_1 = 0$.From this expression we can derive the corresponding Ward identities. Let us consider the correlation function involving a single defect [3],

$$\begin{aligned}
\langle W(\mathcal{C})\rangle &= \int \mathcal{D}\phi W(\mathcal{C}) e^{iS[\phi]} \\
&= \int \mathcal{D}\phi' W'(\mathcal{C}) e^{iS[\phi']} \\
&= \int \mathcal{D}\phi' [W(\mathcal{C}) + \int_{\mathcal{C}} \xi(\Sigma_{D-2})\delta W(\mathcal{C})](1 + i\delta S) e^{iS[\phi']} \\
=> 0 &= \int_{\mathcal{C}} \xi(\Sigma_{D-2})\langle \delta W(\mathcal{C})\rangle + i\langle \delta S\ W(\mathcal{C})\rangle \\
=> 0 &= \int_{\mathcal{C}} \xi(\Sigma_{D-2})\langle \delta W(\mathcal{C})\rangle - i\int d^D x \xi_\nu \partial_\mu \langle J^{\mu\nu} W(\mathcal{C})\rangle
\end{aligned} \tag{45}$$

Here, $\delta S = \int d^D x J^{\mu\nu}\partial_\mu \xi_\nu = -i\int d^D x \xi_\nu \partial_\mu J^{\mu\nu}$. Thus, the above relation enables us to find the Ward identities

$$\begin{aligned}
i\int d^D x \xi_\nu \partial_\mu \langle J^{\mu\nu} W(\mathcal{C})\rangle &= \int_{\mathcal{C}} dy^\nu \xi_\nu \langle \delta W(\mathcal{C})\rangle \\
&= \int d^D x \xi_\nu \int_{\mathcal{C}} dy^\nu \delta^D(x-y)\langle \delta W(\mathcal{C})\rangle \\
=> \partial_\mu \langle J^{\mu\nu} W(\mathcal{C})\rangle &= -i\int_{\mathcal{C}} dy^\nu \delta^D(x-y)\langle \delta W(\mathcal{C})\rangle
\end{aligned} \tag{46}$$

3.3 SSB & Goldstone Mode

We now explore SSB of 1-form symmetries in gauge theories. Since $W[\mathcal{C}]$ is the order parameter for the electric 1-form symmetry, the expectation value, $\langle W(\mathcal{C})\rangle$ usually depends on geometric attributes, such as the area enclosed by C or its perimeter [2]:

$$\langle W[\mathcal{C}]\rangle \sim e^{\text{Area}[\mathcal{C}]} \text{ or } \langle W[\mathcal{C}]\rangle \sim e^{\text{Perimeter}[\mathcal{C}]} \tag{47}$$

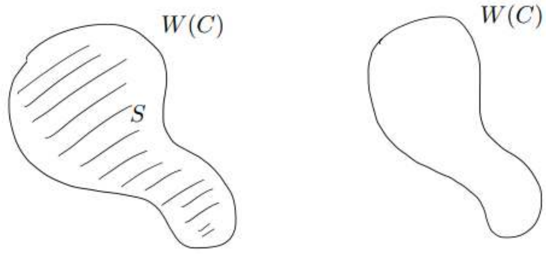

Figure 6: $\langle W[\mathcal{C}]\rangle$ depends on geometric attributes, such as the area enclosed by $\mathcal{C}$ or its perimeter in SSB of a 1-form symmetry.

- **Unbroken Phase:** For a large loop $\mathcal{C}$, the area law decays much more quickly [3]

$$\langle W[\mathcal{C}]\rangle \sim e^{\text{Area}(\mathcal{C})} \to 0 \tag{48}$$

  Hence, the area law is interpreted as a vanishing value of the order parameter, corresponding to an unbroken phase.

  Here, Area$[\mathcal{C}]$ is the area of the minimal surface that fills in the curve $\mathcal{C}$, i.e., the minimal surface S such that $\partial S = \mathcal{C}$. This is the higher-form analogue of the exponential decay shown in 12. To understand this, consider the objects we count – i.e., the string worldsheets – which are massive with tension [5]. Thus, the line operator creates a $2D$ worldsheet that must fill in the curve.

- **SSB Phase:** For a large loop $\mathcal{C}$, the perimeter law almost don't decay [3]

$$\langle W[\mathcal{C}]\rangle \sim e^{\text{Perimeter}(\mathcal{C})} \neq 0 \tag{49}$$

  Therefore, in analogy with the case of 0-form symmetries, the perimeter law is interpreted as implying a nonvanishing value of the order parameter, and then is associated with phases where the 1-form symmetry is spontaneously broken.

  One can imagine this as the strings have "condensed", i.e. that the tension has dropped to zero [5]. The idea here is that just as in 13 the correlation function has factorized as much as possible; however the line operator can always be redefined by a c-number which is a local functional of $\mathcal{C}$, and thus the behavior above is essentially the same as saying that the line operator is a constant which is independent of $\mathcal{C}$. The parameter is thus non-universal.

  If the 1-form symmetry is continuous, this leads to the existence of Goldstone excitations.

To find the Goldstone excitations [3], we can follow precisely the same reasoning we did in the case of 0-form symmetries. The starting point is the Ward identity 46, which we rewrite here with $\langle \delta W(\mathcal{C})\rangle = -iq_e\langle W(\mathcal{C})\rangle$,

$$\partial_\mu \langle J^{\mu\nu} W(\mathcal{C})\rangle = -q_e \int_{\mathcal{C}} dy^\nu \delta^D(x-y)\langle W(\mathcal{C})\rangle \tag{50}$$

Taking the Fourier transform, we get

$$\begin{aligned} ip_\mu \langle J^{\mu\nu}(p) W(\mathcal{C})\rangle &= q_e \int_{\mathcal{C}} dy^\nu e^{ipy} \delta^D(x-y)\langle W(\mathcal{C})\rangle \\ => \lim_{p\to 0} ip_\mu \langle J^{\mu\nu}(p) W(\mathcal{C})\rangle &= q_e \int_{\mathcal{C}} dy^\nu \delta^D(x-y)\langle W(\mathcal{C})\rangle \end{aligned} \tag{51}$$

Whenever $\langle W(\mathcal{C})\rangle \neq 0$, the correlation function $\langle J^{\mu\nu}(p) W(\mathcal{C})\rangle$ must have a pole at $p = 0$. To see this, first note that $\langle J^{\mu\nu}(p) W(\mathcal{C})\rangle \sim c_1(p)[p^\mu \int_{\mathcal{C}} dy^\nu e^{ipy} - p^\nu \int_{\mathcal{C}} dy^\mu e^{ipy}]]$ (obtained by index structure of 2-form current and recalling that it is anti-symmetric in its indices).

Now, in the case of an SSB phase, $\langle \delta\phi_a(p = 0)\rangle \neq 0$ is some constant independent of the momentum p^μ. To satisfy this we must have $c_1(p) \sim \frac{1}{p^2}$ i.e.

$$\langle J^{\mu\nu}(p) W(\mathcal{C})\rangle \sim \frac{p^\mu \int_{\mathcal{C}} dy^\nu e^{ipy} - p^\nu \int_{\mathcal{C}} dy^\mu e^{ipy}}{p^2} \tag{52}$$

This implies that there are massless modes in the spectrum, which is the spontaneous breaking of the continuous 1-form symmetry.

Summary:

	0-form	**1-form**
Current:	$J^\mu = (\rho, \vec{j})$	$J^{\mu\nu} = (J^{0i}, J^{ij})$
Geometry:	pointlike object	stringlike object
Conservation law:	$\partial_\mu J^\mu = 0$	$\partial_\mu J^{\mu\nu} = 0$
Operator:	local operator, $\phi(x)$	line operator, $W(\mathcal{C})$
SSB phase:	$\langle \phi^\dagger(x)\phi(y)\rangle \sim \langle \phi^\dagger(x)\rangle\langle\phi(y)\rangle$	$\langle W(\mathcal{C})\rangle \sim e^{-\text{Perimeter}(\mathcal{C})}$
Unbroken phase:	$\langle \phi^\dagger(x)\phi(y)\rangle \sim e^{-m\|x-y\|}$	$\langle W(\mathcal{C})\rangle \sim e^{-\text{Area}(\mathcal{C})}$

4 Higher form Symmetries in Maxwell Theory

In this section, we will look at $U(1)$ gauge theory with no matter fields i.e., pure Maxwell theory, in the light of generalized symmetries and answer the questions we asked at the beginning.

Consider $A = A_\mu dx^\mu$ be 1-form $U(1)$ gauge field. Its strength is

$$F = dA \tag{53}$$

which is a 2-form on spacetime i.e., $F = \frac{1}{2}F_{\mu\nu}dx^\mu \wedge dx^\nu$. So, the action is given as

$$S = -\frac{1}{2e^2}\int_{\Sigma_D} F \wedge \star F \tag{54}$$

For $U(1)$ symmetry, associated Noether current, $J_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$ such that

$$d \star F = 0 \tag{55}$$

where, Hodge dual $\star F$ is $(D-2)$ -form. Hence, the Noether charge

$$Q_e(\Sigma_{D-2}) = \int_{\Sigma_{D-2}} \star F \tag{56}$$

Now, from 53 and the fact that the exterior derivative operator is nilpotent, i.e, $d^2 = 0$, we get the Bianchi Identity

$$dF = 0 \tag{57}$$

Since Maxwell theory has S-duality (discussed in Appendix A), we can consider F as a Noether current in dual space. Hence, we deduce the existence of a $(D-3)$-form symmetry, for which Noether charge

$$Q_m(\Sigma_2) = \int_{\Sigma_2} F \tag{58}$$

Therefore, we can construct topological operators that enacts this symmetry

$$U_g^{(e)} = e^{i\alpha \int_{\Sigma_{D-2}} \star F} \ \&\ U_g^{(m)} = e^{i\alpha \int_{\Sigma_2} F} \ ;\ g \in U(1) \tag{59}$$

Pure Maxwell theory has $U(1)$ 1-form symmetry, $U_g^{(e)}$ known as the electric 1-form symmetry and $U(1)$ $(D-3)$-form symmetry, $U_g^{(m)}$ known as the magnetic higher form symmetry.

$$U(1)_e^{(1)} \otimes U(1)_m^{(D-3)}$$

4.1 Action on Line Operators

Let's ask, *what are the observables of the theory described by action 54*? An intuitive guess is that it will be gauge-invariant objects, related to $F_{\mu\nu}$ and manifest as local operators.

It turns out that such extended gauge-invariant entities are already well-known in QFT, named Wilson line operators or defects [6]

$$W(q_e, \mathcal{C}) = e^{iq_e \oint_{\mathcal{C}} A} \tag{60}$$

The Wilson line can be interpreted as the world line of a non-dynamical, massive, charged particle. Such a particle is the source of the electric flux. Using insights from [14], we can calculate the correlation of Wilson line

$$\begin{aligned} \langle W(q_e, \mathcal{C})\rangle &= \int \mathcal{D}A e^{iq_e \oint_{\mathcal{C}} A} e^{iS[A]} \\ &= \int \mathcal{D}A \; e^{iq_e \oint_{\mathcal{C}} A - \frac{1}{2e^2}\int_{\Sigma_D} F\wedge\star F} \\ &= \int \mathcal{D}A \; e^{iq_e \int_{\Sigma_D} \delta^{(D-1)}(\mathcal{C})\wedge A - \frac{1}{2e^2} F\wedge\star F} \end{aligned} \tag{61}$$

where, $\delta^{(D-1)}(\mathcal{C})$ is a $(D-1)$-form delta function defined as $\delta^{(D-1)}(\mathcal{C}) = \int_{\Sigma_3^T} \delta^{(D-1)}(\mathcal{C}) = 1$. Here, Σ_{D-1}^T is a $(D-1)$-manifold that transversely intersects $\mathcal{C}$ once.

Hence, it is evident that the Wilson line acts as an electric source, and in its presence, the equation of motion for A be

$$d \star F = q_e e^2 \delta^{(D-1)}(x \in \mathcal{C}) \tag{62}$$

By integrating, we get Link

$$\int_{\Sigma_{D-2}} \star F = q_e e^2 \text{Link}(\Sigma_{D-2}, \mathcal{C}) \tag{63}$$

So, according to 42 we write

$$\langle U_g^e(\Sigma_{D-2}) W[q_e, \mathcal{C}]\rangle = e^{iq_e \alpha \text{Link}(\Sigma_{D-2}, \mathcal{C})} \langle W[q_e, \mathcal{C}]\rangle \tag{64}$$

As we explored S-duality (Appendix A), we can also define a gauge-invariant line operator that is the magnetic dual to the wilson line, named t'Hooft line

$$W(q_m, \mathcal{C}) = e^{iq_m \oint_{\mathcal{C}} B} \tag{65}$$

where B is the dual gauge field, such that $H = \star F = dB$. Hence, we can write

$$\langle U_g^m(\Sigma_{D-2}) W[q_m, \mathcal{C}]\rangle = e^{iq_m \alpha \text{Link}(\Sigma_{D-2}, \mathcal{C})} \langle W[q_m, \mathcal{C}]\rangle \tag{66}$$

Now, due to the presence of charged matter in the theory, the Wilson lines can end on them. Thus, there is a way in which we can unlink Σ_{D-2} and $\mathcal{C}$ (Figure 7), which is the case of trivial linking, meaning $U_e^m(\Sigma_{D-2})$ here essentially becomes the identity operator [13].

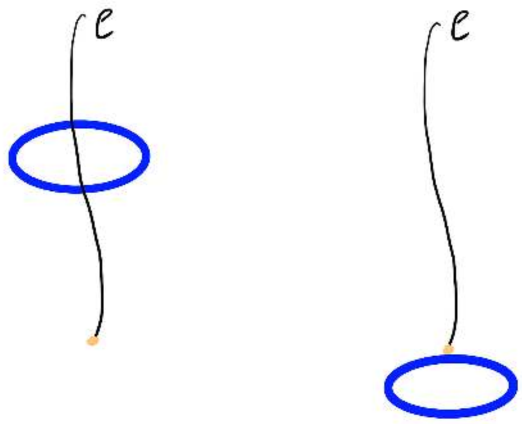

Figure 7: $U_g^e(\Sigma_{D-2})$ wrapping line operator $W[q_e, \mathcal{C}]$ can unlink if $W[q_e, \mathcal{C}]$ end on charges. On the left, we have a non-trivial linking and on the right, a trivial linking. In this case, both sides have to match for $U_g^e(\Sigma_{D-2})$ to be topological, implying $U_g^e(\Sigma_{D-2})$ is the identity operator.

Electric 1-form symmetry is explicitly broken in Maxwell Theory.

On the contrary, no such trivial linking happens for the t'Hooft lines due to the non-existence of a magnetic monopole. This answers our first question, asked at the beginning:

Q. What is the symmetry principle behind the conservation of magnetic flux lines?

Ans: ***Magnetic*** $(D-3)$***-form symmetry*** *is associated with the conservation of magnetic flux lines in Pure Maxwell Theory.*

4.2 Photon as Goldstone Mode !!

In this section, we will discuss the spontaneous breaking of continuous 1-form symmetries [10, 11, 12, 13] of QED in $D = 4$, where the Goldstone modes arising due to the spontaneous breaking of the magnetic 1-form symmetry is precisely the photon. Note that the corresponding conserved current $J_{\mu\nu} \sim \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ creates gapless excitations from the vacuum in the broken phase as

$$|G\rangle \sim J^{\mu\nu}(x)|0\rangle \tag{67}$$

where $|G\rangle$ stands for the state corresponding to the Goldstone mode. Now, the free field expansion in terms of ladder operators satisfying $\partial^2 A^\mu = 0$ (Feynman gauge):

$$A^\mu(x) = \frac{1}{(2\pi)^3}\int \frac{d^3\vec{p}}{\sqrt{2|\vec{p}|}}\sum_{\lambda=1}^{4}\epsilon^\mu_\lambda(p)[a_\lambda(p)e^{-ipx} + a^\dagger_\lambda(p)e^{ipx}] \tag{68}$$

where $\epsilon^\mu_\lambda(p)$ are four linearly independent polarization vectors – out of which not all are physical since some of them do not satisfy $\langle 0|\partial_\mu A^\mu|0\rangle = 0$ (negative norm state) and other states have zero norms. We exclude these to get only two physical polarisations – say corresponding to $\lambda = 1, 2$ in the above equation 68, which is the correct number of physical degrees of freedom for the gauge field A^μ in 4D. Then, a single photon state is created by

$$|\lambda, \vec{p}\rangle = a^\dagger_\lambda(p)|0\rangle;\ \ \lambda = 1, 2 \tag{69}$$

with the commutation relation satisfying

$$[a_\lambda(p), a^\dagger_{\lambda'}(p')] = (2\pi)^3\delta_{\lambda,\lambda'}\delta^{(3)}(\vec{p} - \vec{p'});\ \ \lambda, \lambda' = 1, 2 \tag{70}$$

Now, let us compute $F_{\mu\nu}$ from 68

$$F^{\mu\nu} = \frac{i}{(2\pi)^3}\int \frac{d^3\vec{p}}{\sqrt{2|\vec{p}|}}\sum_{\lambda=1}^{4}\epsilon^\nu_\lambda(p)p^\mu[-a_\lambda(p)e^{-ipx} + a^\dagger_\lambda(p)e^{ipx} - (\mu \longleftrightarrow \nu)] \tag{71}$$

Leading to

$$\langle 0|F^{\mu\nu} = \frac{i}{(2\pi)^3}\int \frac{d^3\vec{p}}{\sqrt{2|\vec{p}|}}\sum_{\lambda=1}^{4}\langle 0|a_\lambda(p)[\epsilon^\mu_\lambda(p)p^\nu - \epsilon^\nu_\lambda(p)p^\mu]e^{-ipx} \tag{72}$$

With the above setup, now let us compute the matrix element between $|\lambda', \vec{p'}\rangle$ and $|G\rangle \sim \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}|0\rangle$:

$$\begin{aligned} \langle G|\lambda, \vec{p}\rangle &= \langle 0|\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}|\lambda', \vec{p'}\rangle \\ &= \frac{i}{(2\pi)^3}\int \frac{d^3\vec{p}}{\sqrt{2|\vec{p}|}}\sum_{\lambda=1}^{4}\epsilon_{\mu\nu\rho\sigma}[\epsilon^\mu_\lambda(p)p^\nu - \epsilon^\nu_\lambda(p)p^\mu]e^{-ipx}\langle 0|a_\lambda(p)a^\dagger_{\lambda'}(p')|0\rangle \\ &= \frac{i}{(2\pi)^3}\int \frac{d^3\vec{p}}{\sqrt{2|\vec{p}|}}\sum_{\lambda=1}^{4}\epsilon_{\mu\nu\rho\sigma}[\epsilon^\mu_\lambda(p)p^\nu - \epsilon^\nu_\lambda(p)p^\mu]e^{-ipx}(2\pi)^3\delta_{\lambda,\lambda'}\delta^{(3)}(\vec{p} - \vec{p'}) \\ &= \frac{i}{\sqrt{2|\vec{p}|}}\epsilon_{\mu\nu\rho\sigma}[\epsilon^\mu_\lambda(p)p^\nu - \epsilon^\nu_\lambda(p)p^\mu]e^{-ipx} \neq 0 \end{aligned} \tag{73}$$

where we have normalised the vacuum as $\langle 0|0\rangle = 1$.

Therefore, We see from that the Goldstone mode has non-vanishing overlap with a single photon state. Thus, we see that the Goldstone excitation is the photon itself.

This is an important fact – the way to think about it is that the vacuum is a sea of "condensed" strings, where the condensed objects are magnetic and electric field lines [5]. Long-wavelength wiggles in this sea of condensed strings are what we call photons.

According to S-duality, which interchanges the Wilson and 't Hooft lines, we can say both higher form symmetries are spontaneously broken in Free Maxwell Theory [5].

So, we got the answer to our second question, asked at the beginning:

Q. Why photons are massless in D = 4?

Ans: *Photons are Goldstone modes for the spontaneous breaking of the continuous* ***magnetic 1-form symmetry*** *associated with the conservation of magnetic flux.*

5 Conclusion

In this report, we have presented a systematic discussion of generalized global symmetries within the framework of quantum field theory, beginning with the familiar case of 0-form symmetries and extending the construction to higher-form symmetries. By revisiting the Noether current, Ward identities, and order parameters from this generalized perspective, we clarified how conserved quantities and topological operators arise naturally as codimension-($p+1$) defects implementing p-form symmetries. We further emphasized how spontaneous symmetry breaking and the appearance of Goldstone excitations generalize beyond the Landau paradigm to encompass extended objects such as strings and surfaces.

Applying these ideas to pure Maxwell theory, we demonstrated that the conservation of magnetic flux lines is a manifestation of a magnetic higher-form symmetry, while the photon in four dimensions can be interpreted as the Goldstone mode of a spontaneously broken continuous 1-form symmetry. This not only offers a symmetry-based explanation for the masslessness of the photon but also illustrates how higher-form symmetries provide a unified language for understanding nontrivial phases and excitations of gauge theories.

Overall, our analysis reinforces the unifying principle

Symmetry in QFTs = Topological operators in QFTs.

This viewpoint captures both conventional and generalized symmetries in a single framework and highlights how topological defects encode the action of symmetries on local and extended operators.

More broadly, the framework of generalized global symmetries is emerging as a powerful tool for organizing and constraining the dynamics of quantum field theories, from high-energy to condensed-matter settings. Its ability to capture phenomena lying beyond conventional order parameters — including topological order, deconfined criticality, and non-Fermi liquids — suggests promising directions for future research. Extending the present analysis to non-Abelian and non-invertible symmetries, as well as to systems with dynamical gravity or holographic duals, may further deepen our understanding of the symmetry structure underlying quantum field theory.

A S-Duality (Electric-Magnetic Duality) in Maxwell Theory

S-Duality in Maxwell theory is the invariance of Maxwell equations under the interchange of electric and magnetic fields. We will explicitly see the calculation below -

Starting with the action of Maxwell's theory is

$$S = \frac{1}{2g}\int_{\mathcal{M}^D} F \wedge \star F \tag{74}$$

where $F = dA$ is $(p+1)$-field strength of p-form potential A. Now, the equation of motion can be obtained by varying with respect to A,

$$\delta S = \frac{1}{g}\int_{\mathcal{M}^D} \delta F \wedge \star F = \frac{1}{g}\int_{\mathcal{M}^D} d(\delta A) \wedge \star F \tag{75}$$

Since $\int d\alpha \wedge \beta = (-1)^{dof(\beta)+1}\int \alpha \wedge d\beta$ then

$$\delta S = (-1)^{p+2}\frac{1}{g}\int_{\mathcal{M}^D} \delta A \wedge d \star F \tag{76}$$

As $\delta S = 0$, hence

$$d \star F = 0 \tag{77}$$

Now, $F = dA$ and the fact that the exterior derivative operator is nilpotent, i.e, $d^2 = 0$, we get the Bianchi Identity

$$dF = 0 \tag{78}$$

To write the action in terms of the dual field strength [15], we introduce an auxiliary independent $(p+1)$-form field U and $(D-p-2)$-form potential B with $(D-p-1)$-form field strength $H = dB$. Hence, the parent action with the Lagrange multiplier term

$$\begin{aligned} S_D &= \frac{1}{2g}\int_{\mathcal{M}^D} U \wedge \star U + (-1)^p \int_{\mathcal{M}^D} dB \wedge U \\ &= \frac{1}{2g}\int_{\mathcal{M}^D} U \wedge \star U + \int_{\mathcal{M}^D} (-1)^p d(B \wedge U) + (-1)^{p+dof(B)+1} B \wedge dU \\ &= \frac{1}{2g}\int_{\mathcal{M}^D} U \wedge \star U + \int_{\mathcal{M}^D} (-1)^p d(B \wedge U) + (-1)^{D-1} B \wedge dU \end{aligned} \tag{79}$$

To realise that this is the same action 74, vary the above action 79 with respect to B, we can identify

$$dU = 0 \text{ so, } U = dA = F \tag{80}$$

Now to obtain the dual action S_D, we again vary the above action with respect to U,

$$\begin{aligned}\frac{\delta S_D}{\delta U} &= 0\\ \Rightarrow \frac{1}{g}\star U + (-1)^p dB &= 0\\ \Rightarrow \star U = -(-1)^p g\, dB &= -(-1)^p gH\\ \Rightarrow U &= -(-1)^p g \star H\end{aligned} \tag{81}$$

Substituting this to the action 79,

$$\begin{aligned}S_D &= \frac{1}{2g}g^2\int_{\mathcal{M}^D}\star H\wedge H - g\int_{\mathcal{M}^D} H\wedge\star H\\ &= \frac{g}{2}\int_{\mathcal{M}^D} H\wedge\star H - g\int_{\mathcal{M}^D} H\wedge\star H\\ &= -\frac{g}{2}\int_{\mathcal{M}^D} H\wedge\star H\end{aligned} \tag{82}$$

Varying the above action 79 with respect to B, we get the equation of motion

$$d\star H = 0 \;\Rightarrow dF = 0;\; H = \star F \tag{83}$$

and like before, Bianchi identity,

$$dH = 0 \;\Rightarrow d\star F = 0 \tag{84}$$

Let's see another action

$$S = \int_{\mathcal{M}^D}\frac{1}{2g}F\wedge\star F + kA\wedge F + \frac{1}{2}G\wedge\star G \tag{85}$$

where $F = dV$ and $G = dA$ are $(p+1)$-field strength of p-form potential V, A respectively. Now, the equation of motion can be obtained by taking variation

$$\begin{aligned}\delta S &= \int_{\mathcal{M}^D}\frac{1}{g}\delta F\wedge\star F + k(\delta A)\wedge F + kA\wedge\delta F + \delta G\wedge\star G\\ &= \int_{\mathcal{M}^D}\frac{1}{g}d(\delta V)\wedge\star F + k(\delta A)\wedge F + kA\wedge d(\delta V) + d(\delta A)\wedge\star G\end{aligned} \tag{86}$$

Collecting the relevant terms with δV,

$$\begin{aligned}\int_{\mathcal{M}^D}\frac{1}{g}d(\delta V)\wedge\star F + kA\wedge d(\delta V) &= \int_{\mathcal{M}^D}\frac{1}{g}d(\delta V)\wedge\star F + (-1)^{2p}k d(\delta V)\wedge A\\ &= \int_{\mathcal{M}^D} d(\delta V)\wedge[\frac{1}{g}\star F + kA]\end{aligned} \tag{87}$$

Since $\int d\alpha\wedge\beta = (-1)^{dof(\beta)+1}\int\alpha\wedge d\beta$ then

$$\int_{\mathcal{M}^D}\delta V\wedge d[(-1)^{D-p-1+1}\frac{1}{g}\star F + (-1)^{p+1}kA] \tag{88}$$

Since the coefficient must vanish for arbitrary dV, thus

$$\begin{aligned} & d[(-1)^{D-p}\frac{1}{g}\star F + (-1)^{p+1}kA] = 0 \\ \Rightarrow\; & d[(-1)^{D-p}\star F + (-1)^{p+1}gkA] = 0 \\ \Rightarrow\; & (-1)^{D-p}d\star(dV) + (-1)^{p+1}gkdA = 0 \end{aligned} \tag{89}$$

Similarly, collecting the relevant terms with δA,

$$\begin{aligned} \int_{\mathcal{M}^D} k(\delta A)\wedge F + d(\delta A)\wedge\star G &= \int_{\mathcal{M}^D} k(\delta A)\wedge F + (-1)^{D-p-1+1}\delta A\wedge d\star G \\ &= \int_{\mathcal{M}^D} \delta A\wedge d[k\star F + (-1)^{D-p}d\star G] \end{aligned} \tag{90}$$

Since the coefficient must vanish for arbitrary dA, thus

$$\begin{aligned} & d[(-1)^{D-p}d\star G + k\star F] = 0 \\ \Rightarrow\; & (-1)^{D-p}d\star(dA) + kdV = 0 \end{aligned} \tag{91}$$

To write the action in terms of the dual field strength [15], we introduce an auxiliary independent $(p+1)$-form field U and 0-form potential ϕ as Lagrange multiplier to enforce $U = dV = F$. Hence, the parent action with the Lagrange multiplier term

$$S_D = \int_{\mathcal{M}^D} \frac{1}{2g}U\wedge\star U + kA\wedge U + \frac{1}{2}G\wedge\star G + d\phi\wedge U \tag{92}$$

To realize that this is the same action 85, vary the above action 92 with respect to ϕ, we can identify

$$dU = 0 \text{ so, } U = dV = F \tag{93}$$

Now to obtain the dual action S_D, we again vary the above action with respect to U,

$$\begin{aligned} & \frac{\delta S_D}{\delta U} = 0 \\ => & \frac{1}{g}\star U + kA + d\phi = 0 \\ => & \star U = -g[kA + d\phi] = -gH \\ => & U = -g\star H \end{aligned} \tag{94}$$

Substituting this to the action 92,

$$\begin{aligned} S_D &= \int_{\mathcal{M}^D} \frac{1}{2g}g^2\star H\wedge H + \frac{1}{2}G\wedge\star G - gH\wedge\star H \\ &= \int_{\mathcal{M}^D} \frac{g}{2}H\wedge\star H + \frac{1}{2}G\wedge\star G - gH\wedge\star H \\ &= \int_{\mathcal{M}^D} -\frac{g}{2}H\wedge\star H + \frac{1}{2}G\wedge\star G \end{aligned} \tag{95}$$

Varying the above action 92 with respect to ϕ,

$$\delta H = d(\delta\phi) \tag{96}$$

Hence,

$$\begin{aligned} 0 = \delta S_D &= \int_{\mathcal{M}^D} \frac{g}{2} d(\delta\phi) \wedge \star H \\ &= (-1)^{D-p-1+1} \frac{g}{2} \int_{\mathcal{M}^D} \delta\phi \wedge d \star H \end{aligned} \tag{97}$$

Therefore,

$$d \star H = 0 \;\Rightarrow d \star [kA + d\phi] = 0 \tag{98}$$

Varying the above action 92 with respect to A,

$$\delta H = k(\delta A), \delta G = d(\delta A) \tag{99}$$

Hence,

$$\begin{aligned} 0 = \delta S_D &= \int_{\mathcal{M}^D} \frac{g}{2} k(\delta A) \wedge \star H + \frac{1}{2} d(\delta A) \wedge \star G \\ &= \int_{\mathcal{M}^D} \frac{g}{2} k(\delta A) \wedge \star H + (-1)^{D-p-1+1} \frac{1}{2} \delta A \wedge d \star G \\ &= \int_{\mathcal{M}^D} \delta A \wedge \frac{1}{2} [gk \star H + (-1)^{D-p} d \star G] \end{aligned} \tag{100}$$

Therefore,

$$(-1)^{D-p} d \star G + k \star H = 0 \;\Rightarrow (-1)^{D-p} d \star (dA) + k \star [kA + d\phi] = 0 \tag{101}$$

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INTERNSHIP AGREEMENT

This Internship Agreement (“Agreement”) is made on this 15th July, between:

- **ICTP PWF: Bangladesh**, represented by Prof. Nabil Iqbal and Ahmed Rakin Kamal (Program Organizers),
- **Mentor, [Mentor Name],** Dr. Arpit Das
- **Intern, [Intern Name]**, Md. Samim ul Islam

Collectively referred to as the “Parties.”

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1. Purpose

The purpose of this Agreement is to set out the terms under which the Intern will participate in the ICTP PWF: Bangladesh online summer mentorship program for physics and/or mathematics research (“Internship”).

2. Duration

- **Start Date:** 15th July 2025
- **End Date:** 15th October 2025
- **Total Duration:** 12 weeks

3. Duties & Responsibilities

3.1 Intern

- Attend all scheduled online meetings and training sessions.
- Complete assigned readings, exercises, and research tasks under the guidance of the mentor.
- Prepare and submit a **Short Report** (2–10 pages) summarizing work progress, results, and reflections by the final day of the Internship.

3.2 Mentor

- Provide regular guidance, feedback, and resources to facilitate the Intern’s learning.

- Assess the Intern's progress at mid-term and final stages.
- Confirm whether the Intern has achieved "considerable progress" toward the research goals outlined at the start.

4. Certificate of Completion

A **Certificate of Completion** will be awarded to the Intern **only if**:

1. The Mentor formally confirms, at the end of the Internship, that the Intern has demonstrated **considerable progress** in their assigned work; and
2. The Intern has submitted the required **Short Report** by the deadline.

No certificate will be issued if either condition is not satisfactorily met.

5. No Monetary Exchange

- **No Stipend for Intern:** The Intern acknowledges that this Internship is purely educational and that ICTP PWF: Bangladesh or the Mentor will **not** provide any monetary compensation, stipend, or benefits.
- **No Fees from Intern:** The Intern will **not** be required to pay any tuition fee, application fee, or other charges to ICTP PWF: Bangladesh, organizers, or Mentors.
- Both Parties understand and agree that this is a volunteer mentorship arrangement with **no financial obligations** on either side.

6. Intellectual Property

- Unless otherwise agreed in writing, any original work (data, code, reports) produced by the Intern during the Internship remains the property of the Intern and the Mentor. ICTP PWF Bangladesh should be **acknowledged** in any research articles.
- The Intern grants ICTP PWF: Bangladesh and the Mentor a **non-exclusive**, **royalty-free** license to use, reproduce, and display the Short Report and any related presentation materials for educational and promotional purposes.

7. Confidentiality

The Intern agrees to keep confidential any unpublished data, methodologies, or proprietary information disclosed by the Mentor or ICTP PWF: Bangladesh during the Internship, both during and after the term of this Agreement.

8.Appropriate Conduct:

- Bullying, harassment, or intimidation in any form—verbal, physical, or cyber—will not be tolerated by either the mentor or the mentee. Violations must be reported to the organisers.
- Mutual respect and adherence to moral values are expected at all times.

9. Termination

- Either Party may terminate this Agreement with **seven (7) days' written notice** if the other Party breaches any material term and fails to remedy within that notice period.

- Upon termination, the Intern will cease all Internship activities and return any confidential materials.

Signatures

Prof. Nabil Iqbal
Program Organizer, ICTP PWF Physics for Bangladesh

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Ahmed Rakin Kamal
Program Organizer, ICTP PWF Physics for Bangladesh

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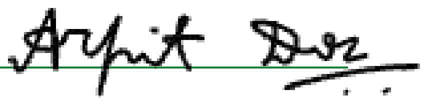
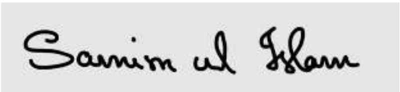
[Mentor Name and Institution]

Arpit Das, University of Edinburgh



[Intern Name and Institution]

Samim ul Islam, Dhaka University



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Date: 15th July 2025